Chapter 3

Flow Analysis

3.1 Introduction

Pipes and ducts are the veins and arteries of mechanical systems such as a power-plants, refineries, or HVAC systems. Without them, these systems could not exist. As in our own bodies, where the veins and arteries move blood through the pumping action of the heart, power plants require the circulation of a "working" fluid in order to provide its functionality. In this chapter we will examine the basics of flow analysis and piping system design, and develop the framework for the analysis of integrated mechanical systems such as powerplants, refineries, and airflow systems, to name just a few.

3.2 Flow in Pipes and Channels

Analysis of flow in piping systems always begins with some form of the mechanical energy equation. Initially, we may consider the Bernoulli equation:

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \tag{3.1}$$

where $\gamma = \rho g$.

However, our intuition will tell us that losses throughout the system will limit application of Eq. (3.1). We may however use Eq. (3.1) to determine system behaviour under ideal comditions, and use these results as a target for maximum performace. In order to apply Eq. (3.1) to a piping system, we must extend the Bernoulli equation to account for losses which result from pipe fittings, valves, and direct losses (friction) within the pipes themselves. The extended Bernoulli equation may be written as:

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_{losses} \tag{3.2}$$

Additionally, at various points along the piping system we may need to add energy to provide an adequate flow. This is generally achieved through the use of some sort of prime mover, such as a pump, fan, or compressor. For a system containing a pump or pumps, we must include an additional term to account for the energy supplied to the flowing stream. This yields the following form of the energy equation:

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + \sum \frac{\dot{W}_{pumps}}{\dot{m}g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_{losses} \tag{3.3}$$

Finally, if somewhere in the piping system a component extracts energy from the fluid stream, such as a turbine, the energy equation takes the form:

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + \sum \frac{\dot{W}_{pumps}}{\dot{m}g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum \frac{\dot{W}_{turbines}}{\dot{m}g} + \sum h_{losses} \tag{3.4}$$

This statement of the mechanical energy equation from fluid dynamics states that the initial energy at point 1 plus any energy supplied by a pump or pumps must balance with the energy at point 2 plus any losses incurred due to fluid friction, pipe fittings, valves etc., plus any energy output through mechanical conversion. In a fluid piping system, whether we are dealing with a single pipe or a network of pipes, Eqs. (3.1-3.4) will be at the center of analysis. Note, that Eq. (3.4) is written to reflect that all work terms have a *positive* sign convention. In the classic formulation of the energy equation work done on a system and work done by a system have different signs. By isolating each component of shaft work separately we have included the sign convention by appropriately placing the work terms on the left side for work done on a system and the right side for work done by the system. This is best summarized by Fig. 3.1. The student should also take note that headloss represents a loss in total head or pressure and not merely the loss in static pressure, i.e.

$$\left(\frac{P_1}{\gamma} + \frac{V_1^2}{2g}\right) - \left(\frac{P_2}{\gamma} + \frac{V_2^2}{2g}\right) + (z_1 - z_2) = \sum h_{losses} \tag{3.5}$$

It is evident from Fig. 3.1, that static pressure recovery occurs whenever the flow velocity decreases, but that total pressure drop still diminishes.

3.3 Losses in Piping Systems

Losses in a piping system are typically categorized as major and minor losses. Minor losses in piping systems are generally characterized as any losses which are due to pipe inlets and outlets, fittings and bends, valves, expansions and contractions, filters and screens, etc. Essentially, everything within the system which is not a section of pipe or other major component. In fact, in many flow systems the minor losses can account for more head loss, than the pipes themselves. The primary distinction comes in the manner in which major and minor losses are calculated.

In the mechanical energy equation, head losses are computed from the following expression:

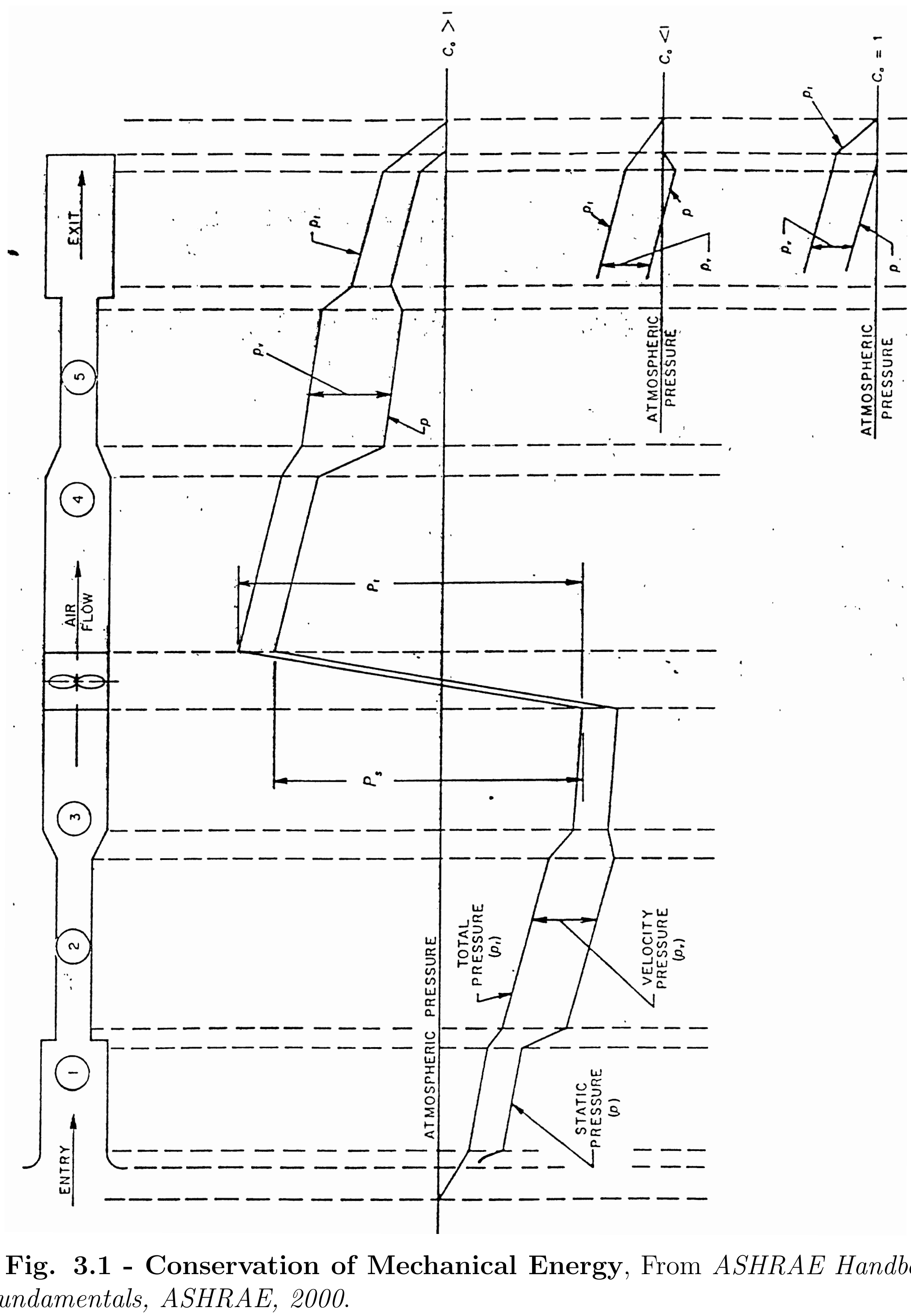


Fig. 3.1 - Conservation of Mechanical Energy, From *ASHRAE Handbook - Fundamentals, ASHRAE, 2000.*

$$h_L = \sum \frac{4 f_i L_i}{D_i} \frac{\overline{V}_i^2}{2g} + \sum K_i \frac{\overline{V}_i^2}{2g} + \sum h_{comp} \tag{3.6}$$

or in terms of pressure loss,

$$\Delta p_L = \sum \frac{4 f_i L_i}{D_i} \frac{\rho \overline{V}_i^2}{2} + \sum K_i \frac{\rho \overline{V}_i^2}{2} + \sum \Delta p_{comp} \tag{3.7}$$

Care must be taken, that the appropriate mean flow velocity is used in each term for each individual length of pipe and each minor loss. The third group represents major losses due to components within a system that the fluid must flow through. Since velocity can vary through a flow system, it is highly recommended to define each velocity in terms of the system mass flow rate $\dot{m} = \rho \overline{V}_i A_i$.

3.3.1 Minor Losses

Minor losses in systems are most often calculated using the concept of a loss coefficient or equivalent friction length method. In this chapter we exclusively use the the K-factor or loss coefficient method. In the loss coefficient method, a constant or variable factor K is defined as:

$$K = \frac{h_f}{V^2/2g} = \frac{\Delta P}{\frac{1}{2}\rho V^2} \tag{3.8}$$

The associated head loss is related to the loss coefficient through

$$h_f = K \frac{V^2}{2g} \tag{3.9}$$

Values of K have been determined experimentally for many configurations of bends, fittings, valves etc. A comprehensive source of this loss coefficient data is found in the excellent reference *Handbook of Hydraulic Resistance* by Idelchik (1986) or the *Applied Fluid Dynamics Handbook* by Blevins (1984). These books are now considered the leading source of design data for hydraulic systems. Some useful expressions for the computing minor losses in pipe systems are supplied in the figures below. Figure 3.2 provides simple constant K factors, whereas Figs. 3.3a-3.3c provide K factors which are dependent on specific geometric details.

We conclude with two useful expressions which are used often in flow modelling, the sudden contraction loss and the sudden expansion loss. These may be predicted using the simple relationships:

$$K_{SC} \approx 0.42(1 - \sigma^2)^2 \tag{3.10}$$

and

$$K_{SE} \approx (1 - \sigma)^2 \tag{3.11}$$

Type of fitting or valve	Loss coefficient (K)
45° elbow, standard	0.35
45° elbow, long radius	0.2
90° elbow, standard	0.75
Long radius	0.45
Square or miter	1.3
180° bend, close return	1.5
Tee, standard, along run, branch blanked off	0.4
Used as elbow, entering run	1.3
Used as elbow, entering branch	1.5
Branching flow	1.0
Coupling	0.04
Union	0.04
Gate valve, open	0.17
$\frac{3}{4}$ open	0.9
$\frac{1}{2}$ open	4.5
$\frac{1}{4}$ open	24.0
Diaphragm valve, open	2.3
$\frac{3}{4}$ open	2.6
$\frac{1}{2}$ open	4.3
$\frac{1}{4}$ open	21.0
Globe valve, bevel seat, open	6.4
$\frac{1}{2}$ open	9.5
Composition seat, open	6.0
$\frac{1}{2}$ open	8.5
Plug disk, open	9.0
$\frac{3}{4}$ open	13.0
$\frac{1}{2}$ open	36.0
$\frac{1}{4}$ open	112.0
Angle valve, open	3.0
Y or blowoff valve, open	3.0
Plug cocks, θ = 5°	0.05
10°	0.29
20°	1.56
40°	17.3
60°	206.0
Butterfly valve, θ = 5°	0.24
10°	0.52
20°	1.54
40°	10.8
60°	118.0
Check valve, swing	2.0
Disk	10.0
Ball	70.0
Foot valve	15.0
Water meter, disk	7.0
Piston	15.0
Rotary (star-shaped disk)	10.0
Turbine wheel	6.0

Used with permission, from J.H. Perry and C.H. Chilton, *Chemical Engineers' Handbook*, McGraw-Hill Book Company, 1963.

Fig. 3.2 - Typical Constant K-Factors.

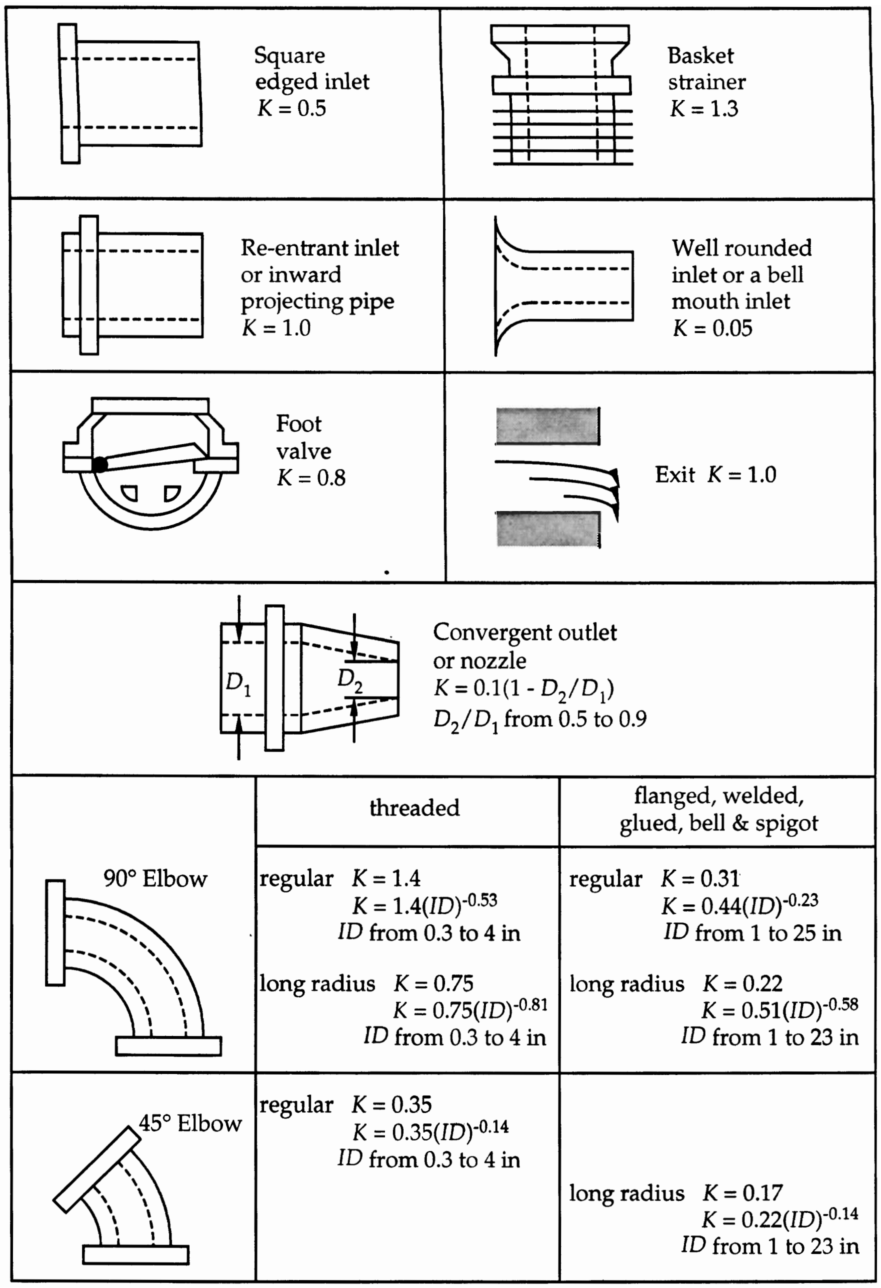


Fig. 3.3a - Variable K-Factors, From *Design of Fluid-Thermal Systems, Janna*, PWS Publishing, 1998.

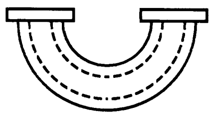
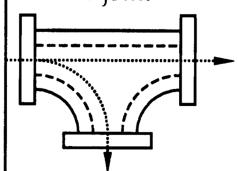
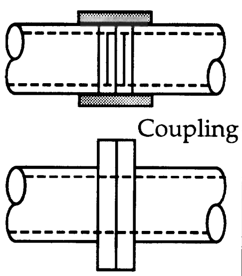
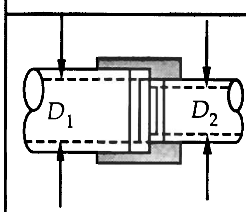
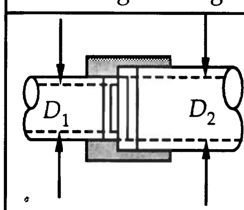
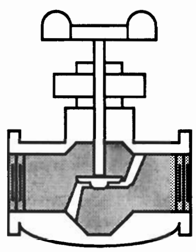
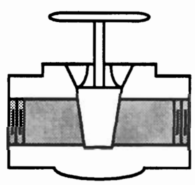
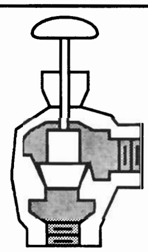
	threaded	flanged, welded, glued, bell & spigot
Return bend 	regular $K = 1.5$ $K = 1.5(ID)^{-0.57}$ ID from 0.3 to 4 in	regular $K = 0.3$ $K = 0.43(ID)^{-0.26}$ ID from 1 to 23 in long radius $K = 0.2$ $K = 0.43(ID)^{-0.53}$ ID from 1 to 23 in
T joint 	line flow $K = 0.9$ all sizes ID from 0.3 to 4 in branch flow $K = 1.9$ $K = 1.9(ID)^{-0.38}$ ID from 0.3 to 4 in	line flow $K = 0.14$ $K = 0.27(ID)^{-0.46}$ ID from 1 to 20 in branch flow $K = 0.69$ $K = 1.0(ID)^{-0.29}$ ID from 1 to 20 in
 Coupling	$K = 0.08$ $K = 0.083(ID)^{-0.69}$ ID from 0.4 to 4 in	$K = 0.08$ ID from 0.3 to 23 in
 Reducing bushing	$K = 0.5 - 0.167(D_2/D_1) - 0.125(D_2/D_1)^2 - 0.208(D_2/D_1)^3$ $0.25 < D_2/D_1 < 1$	
 Sudden expansion	$K = ((D_2/D_1)^2 - 1)^2$ $1 < D_2/D_1 < 5$	

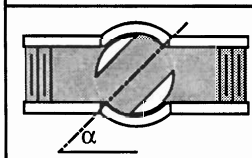
Fig. 3.3b - Variable K-Factors, From *Design of Fluid-Thermal Systems, Janna*, PWS Publishing, 1998.

	threaded	flanged, welded, glued, bell & spigot
Globe valve 	fully open $K = 10$ $K = \exp\{2.158 - 0.459 \ln(ID) + 0.259[\ln(ID)]^2 - 0.123[\ln(ID)]^3\}$ ID from 0.3 to 4 in	fully open $K = 10$ $K = \exp\{2.565 - 0.916 \ln(ID) + 0.339[\ln(ID)]^2 - 0.01416[\ln(ID)]^3\}$ ID from 0.3 to 4 in
Gate Valve 	fully open $K = 0.15$ $K = 0.24(ID)^{-0.47}$ ID from 0.3 to 4 in	fully open $K = 0.15$ $K = 0.78(ID)^{-1.14}$ ID from 1 to 20 in

Gate Valve — All sizes

Fraction closed	0	1/4	3/8	1/2	5/8	3/4	7/8
K =	0.15	0.26	0.81	2.06	5.52	17.0	97.8

	threaded	flanged, welded, glued, bell & spigot
 Angle Valve	fully open $K = 2.0$ $K = 4.5(ID)^{-1.08}$ ID from 0.6 to 4 in	fully open $K = 2.0$ $K = \exp\{1.569 - 1.43 \ln(ID) + 0.8[\ln(ID)]^2 - 0.137[\ln(ID)]^3\}$ ID from 1 to 20 in



Ball Valve — All sizes

α° =	0	10	20	30	40	50	60	70	80
K =	0.05	0.29	1.56	5.47	17.3	25.6	206	485	∞

Check Valves	threaded	flanged, welded, glued, bell & spigot
Swing Type	$K = 2.5$	$K = 2.5$
Ball Type	$K = 70.0$	$K = 70.0$
Lift Type	$K = 12.0$	$K = 12.0$

Fig. 3.3c - Variable K-Factors, From *Design of Fluid-Thermal Systems, Janna*, PWS Publishing, 1998.

where

$$0 < \sigma = \frac{A_1}{A_2} < 1 \tag{3.12}$$

Here A_1 and A_2 denote the smaller and larger duct areas, respectively. In the above expressions the K-factor is based on the velocity through the smaller section. In the preceding figures, the K-factors are based on the velocity of the flow through the piping element. In the case of branching components, the K-factor is usually based on the flow entering the pipe element for flow splitting device, and the flow leaving for a flow combining device.

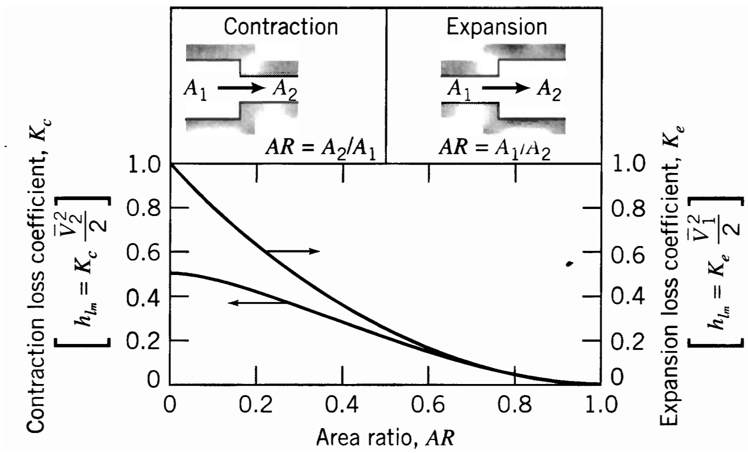


Fig. 3.4 - Sudden Expansion/Contraction Losses, From *Introduction to Fluid Mechanics, Fox and McDonald*, Wiley, 2006.

3.3.2 Major Losses

Major losses of head in a piping system are the direct result of fluid friction in pipes and ducting. The resulting head losses are usually computed through the use of friction factors. Friction factors for ducts have been compiled for both laminar and turbulent flows. Two widely adopted definitions of the friction factor are the *Fanning* friction factor defined as:

$$f_F = \frac{\overline{\tau}}{\frac{1}{2}\rho V^2} \tag{3.13}$$

and the *Darcy* friction factor defined as:

$$f_D = \frac{-\frac{dp}{dx}D}{\frac{1}{2}\rho V^2} \tag{3.14}$$

Now for a circular pipe, the shear stress $\overline{\tau}$, is related to the pressure gradient dp/dx, by means of the following relation derived from a force balance on an element of fluid

$$\overline{\tau} = -\frac{A}{P}\frac{dp}{dx} = -\frac{D_h}{4}\frac{dp}{dx} \tag{3.15}$$

This leads to the definition of the hydraulic diameter and results in the following relationship between the two definitions:

$$f_D = 4 f_F \tag{3.16}$$

In this chapter we will use exclusively use the *Fanning* friction factor definition. Therefore, the head loss characteristic of a circular pipe may be written as:

$$h_f = 4\frac{f_F L}{D_h}\frac{V^2}{2g} \tag{3.17}$$

In future we will drop the subscript F and assume explicitly the definition of the *Fanning* friction factor unless otherwise indicated. Note that Eqs. (3.5) and (3.6) are written in terms of the Fanning friction factor.

For a circular duct three different friction factor models are of interest. These are: fully developed laminar flow, developing laminar flow, and fully developed turbulent flow. The following design correlations for the friction factor are of interest for circular pipes and non-circular ducts.

Fully Developed Laminar Flow, $Re_{D_h} < 2300$

In the fully developed laminar flow region in a circular tube, the *Fanning* friction factor which is often written in terms of the friction factor-Reynolds number product is:

$$f Re_{D_h} = 16 \tag{3.18}$$

or

$$f = \frac{16}{Re_{D_h}} \tag{3.19}$$

In many engineering systems, flow in a duct which is not circular in cross-section is quite common. The definitions above are still applicable through the definition of the hydraulic diameter. The hydraulic diameter arises from Eq. (3.14). For a non-circular duct or channel the hydraulic diameter is defined as

$$D_h = \frac{4A_c}{P} \tag{3.20}$$

where A_c is the cross-sectional area and P is the wetted perimeter of the non-circular duct.

Equation (3.19) has been adopted by virtually all hydraulics engineers. As we shall see, the concept of the hydraulic diameter is quite useful in applications where the flow is turbulent, since the turbulent friction factor is only weakly dependent upon duct shape. This is not the case when the flow is laminar. Laminar flow friction factors are a strong function of duct shape. As a result, we must be careful when approximating friction factors for non-circular ducts. As we saw earlier in Chapter 2, for non-circular ducts the laminar friction factor is obtained from:

$$f = \frac{C}{Re_{D_h}} \tag{3.21}$$

where the constant C is often tabulated (see Chapter 2) for specific duct shapes. It is evident that the hydraulic diameter concept does not allow for a single value of the friction factor to be used for laminar flow in non-circular ducts. All of the results of Table 2 in Chapter 2 can easily be predicted from the following expression which is derived from the analytical solution for the rectangular duct:

$$fRe_{D_h} = \frac{12\beta^{3/2}}{(1+\beta)\left[1 - \dfrac{192}{\pi^5\beta}\tanh\left(\dfrac{\pi}{2}\beta\right)\right]}\left(\frac{4\sqrt{A_c}}{P}\right) \tag{3.22}$$

where $1 < \beta < \infty$ is the duct aspect ratio, A is the cross-sectional area, and P is the perimeter of the non-circular duct. Note, that $\beta = 100$ is a fair approximation to the parallel plate channel. This expression may be used to predict the fRe_{D_h} value for many other non-circular cross-sections within $\pm$ 10 percent, provided that the smallest re-entrant corner angle is greater than 15 degrees. The aspect ratio β for most non-circular ducts is simply a measure of the slenderness of the duct.

Developing Laminar Flow, $Re_{D_h} < 2300$ **and** $L < L_e$

In the laminar entrance region, the pressure drop is much larger owing to the development of the hydrodynamic boundary layers, see Fig. 3.4. If a tube is short or not much longer than the region where boundary layers develop an alternate approach is required to predict the pressure drop. The entrance length is defined such that

$$L_e \approx 0.05 D_h Re_{D_h} \tag{3.23}$$

where

$$\begin{array}{ll} L << L_e & \text{Short Duct} \\ L >> L_e & \text{Long Duct} \end{array} \tag{3.24}$$

A simple model developed by Muzychka (1999) for a duct of any length L is:

$$f_{app}Re_{D_h} = \left[\left(\frac{3.44}{\sqrt{L^*}}\right)^2 + \left(fRe_{D_h}\right)^2\right]^{1/2} \tag{3.25}$$

where the apparent friction factor f_{app} combines the effects of the wall shear stress and increase in pressure drop due to the accelerating core in the entrance region. The dimensionless parameters L^* and Re_{D_h} are defined as:

$$L^* = \frac{L}{D_h Re_{D_h}} \quad Re_{D_h} = \frac{\rho V D_h}{\mu} \tag{3.26}$$

The remaining parameter fRe_{D_h} is the fully developed friction factor-Reynolds number product defined earlier. The pressure drop in terms of the mass flow rate and the fRe group is obtained from the following equation:

$$\Delta p = \frac{2(f_{app}Re_{D_h})\dot{m}\nu L}{D_h^2 A_c} \tag{3.27}$$

For developing laminar flows Eq. (3.24) is still applicable since the term which accounts for the entrance region pressure drop, is independent of duct shape. On the other hand as we have seen, the second term which accounts for the fully developed flow component, is a strong function of duct shape. Therefore, one merely has to substitute the appropriate value of the fRe_{D_h} group for the duct of interest. Equation (3.24) provides accuracy within ± 5 percent for most common duct shapes.

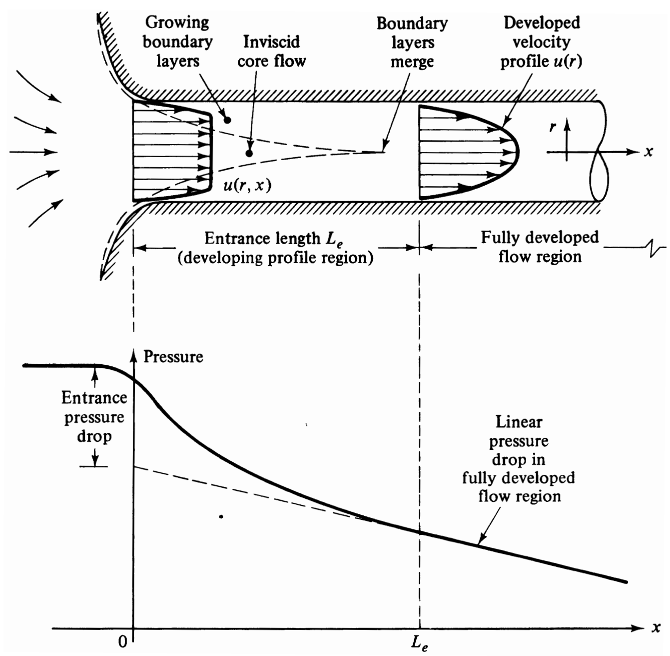


Fig. 3.5 - Laminar Development of Velocity in a Circular Pipe, From *Fluid Mechanics, White*, McGraw-Hill, 2000.

Fully Developed Turbulent Flow, $Re_{D_h} > 4000$

Turbulent duct flows have been traditionally analyzed using empirical methods. Numerous friction factor models have been developed for both smooth and rough walled ducts. The earliest models for smooth ducts are the Blasius and Prandtl models. While for rough ducts, the early studies of Nikuradse, Colebrook, and Von Karman, eventually lead the way for development of the well known Moody diagram shown below. More recently, a number of explicit equations have been developed for eliminating the need for the Moody diagram.

Turbulent friction in smooth pipes has received considerable attention due to the large number of applications. Numerous models have been proposed for both smooth and rough pipes. We consider only the most popular and/or robust models. These models which span some eight decades are found in the numerous heat transfer and fluid dynamics handbooks. Several are given below.

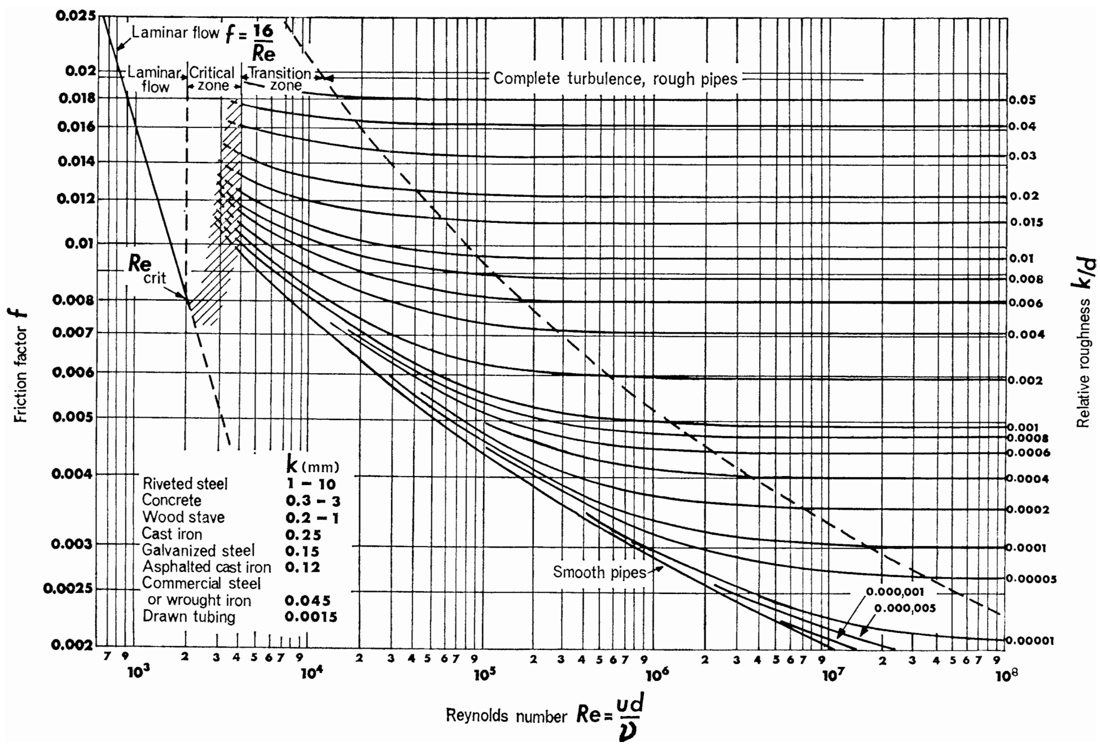


Fig. 3.6 - The Moody Diagram, From *Mechanics of Fluids, Massey*, Van Nostrand Reinhold, 1975.

Blasius Model

The earliest model for turbulent friction in a smooth pipe is attributed to Blasius (1911). He analysed numerous published data and non-dimensionalized them using Reynolds' similarity law, i.e. the Reynolds number. His empirical formulation which is valid for $4000 < Re_{D_h} < 100,000$ is:

$$f = \frac{0.0791}{Re_{D_h}^{1/4}} \tag{3.28}$$

The above formula represented the available data at that time quite well. Unfortunately, no data were available to Blasius for Reynolds numbers greater than 100,000. Subsequently, it was found the model provided poor agreement outside of this Reynolds number range. The model is still useful for problems where the Reynolds number is in range, due to its simplicity and accuracy.

Prandtl-Karman-Nikuradse Model

Another popular expression for fluid friction in smooth pipes was developed by the early pioneers, Prandtl, Karman, and Nikuradse. In some texts it is referred to as the PKN model. It has a semi-theoretical basis and is valid for Reynolds numbers much larger than permitted in the Blasius model. The PKN model has the form:

$$\frac{1}{\sqrt{f}} = 1.7372 \ln(Re_{D_h}\sqrt{f}) - 0.3946 \tag{3.29}$$

It is valid for Reynolds numbers in the range $4000 < Re_{D_h} < 10^7$, and agrees with the experimental data of Nikuradse and others within ± 2 percent. However, due to its implicit nature, it is not very useful.

Early experiments by Nikuradse with sand grains, showed that roughness was a factor in turbulent flows. Later, Colebrook, showed that commercial pipe roughness behaved in a different manner than sand grain roughness for turbulent flows with moderate Reynolds numbers. At large values of the Reynolds number, where the friction factor becomes a constant, both sand grain and commercial pipe roughness behave in a similar manner. Fig. 3.7 provides typical roughness values for commercial pipe materials.

Pipe Material	ε, ft	ε, cm
Steel		
Commercial	0.00015	0.004 6
Corrugated	0.003–0.03	0.09–0.9
Riveted	0.003–0.03	0.09–0.9
Galvanized	0.0002–0.0008	0.006–0.025
Mineral		
Brick sewer, Cement–asbestos, Clays, Concrete	0.001–0.01	0.03–0.3
Wood stave	0.0006–0.003	0.018–0.09
Cast iron	0.00085	0.025
Asphalt coated	0.0004	0.012
Bituminous lined	0.000008	0.000 25
Cement lined	0.000008	0.000 25
Centrifugally spun	0.00001	0.000 31
Drawn tubing	0.000005	0.000 15
Miscellaneous		
Brass, Copper, Glass, Lead, Plastic, Tin	0.000005	0.000 15
Galvanized	0.0002–0.0008	0.006–0.025
Wrought iron	0.00015	0.004 6
PVC	Smooth	Smooth

Fig. 3.7 - Commercial Pipe Roughness, From *Mechanics of Fluids, Massey*, Van Nostrand Reinhold, 1975.

Colebrook and White Model

Fully developed turbulent flow in rough ducts may be characterized by the Colebrook expression which is the basis for the turbulent portion of the *Moody* diagram:

$$\frac{1}{\sqrt{f_d}} = -2\log\left(\frac{k/D_h}{3.7} + \frac{2.51}{Re_{D_h}\sqrt{f_d}}\right) \tag{3.30}$$

This expression, although highly accurate, is not very amenable to design due to its implicit nature. However, for fully rough pipe flows where the friction factor is independent of Reynolds number, the simpler expression:

$$\frac{1}{\sqrt{f_d}} = -2\log\left(\frac{k/D_h}{3.7}\right) \tag{3.31}$$

finds many uses in modelling.

Moody Model

Moody (1944) proposed the following explicit formula for predicting turbulent rough surface friction factors:

$$f = 0.001375\left[1 + \left(20,000\frac{k}{D_h} + \frac{10^6}{Re_{D_h}}\right)^{1/3}\right] \tag{3.32}$$

Massey (1975) reports that the above equation provides ±5% accuracy as compared with the Colebrook and White model, however, Haaland (1983) reports errors as large as 16 %.

Swamee and Jain Model

An alternate form of the turbulent friction model proposed by Swamee and Jain (1976) which provides accuracy within ±1.5% is given by:

$$f = \frac{1}{16\left[\log\left(\dfrac{k/D_h}{3.7} + \dfrac{5.74}{Re_{D_h}^{9/10}}\right)\right]^2} \tag{3.33}$$

Haaland (1983) contends that the accuracy of the above model is actually ±3% when compared with the results computed using the Colebrook and White model.

Haaland Model

The most recent of the explicit friction factor models is due to Haaland (1983). Haaland proposed for systems with a relative roughness $k/D_h > 10^{-4}$:

$$f = \frac{0.3086}{\left\{2\log\left[\frac{6.9}{Re_{D_h}} + \left(\frac{k/D_h}{3.7}\right)^{1.11}\right]\right\}^2} \tag{3.34}$$

Haaland's justification for formulating yet another explicit model, was that none of the above explicit models were simple and accurate. These grounds are easily questioned on the basis that the Colebrook and White model represents data within a margin of error larger than ±5%, in any case. Further, there is not much difference in the expressions 3.32 and 3.33.

Churchill Model

Presently, there is only one model which represents the complete Moody diagram. This model, which was developed by Churchill (1977), combines the laminar and turbulent regions with a linearly interpolated transition region. Since the model is a representation of the Moody diagram, it is applicable only to circular pipes. A correlation of the *Moody* diagram was developed by Churchill (1977). It spans the entire range of laminar, transition, and turbulent flow in pipes. It consists of the following expressions:

$$f = 2\left[\left(\frac{8}{Re_{D_h}}\right)^{12} + \left(\frac{1}{(A_1 + A_2)^{3/2}}\right)\right]^{1/12} \tag{3.35}$$

where

$$A_1 = \left\{2.457\ln\left[\frac{1}{(7/Re_{D_h})^{0.9} + (0.27k/D_h)}\right]\right\}^{16} \tag{3.36}$$

and

$$A_2 = \left(\frac{37530}{Re_{D_h}}\right)^{16} \tag{3.37}$$

The present model is applicable only to circular pipes and is a mathematical representation of the Moody diagram. Results in the transition region must be used with caution, as a linear interpolation of the approximate trend of data is constructed.

Flow Through Porous Media

In many engineering systems, filtration beds or packed columns are used in various processes. These components consist of a channel or duct which contains some form of porous material or a collection of randomly packed spheres or other non-spherical particle. A simple model for predicting pressure drop through packed columns was developed by Ergun (1952). The model which is now commonly referred to as the Ergun equation is a composite solution containing two asymptotic results. One is for slow viscous flow and the other for highly turbulent flow. The resulting model takes the following form:

$$f = \frac{(\Delta p/L)}{\frac{1}{2}\rho V_o^2} \frac{D_p}{4} = \frac{75}{Re_{D_p}} \frac{(1-\epsilon)^2}{\epsilon^3} + \frac{1.75}{2} \left(\frac{1-\epsilon}{\epsilon^3} \right) \tag{3.38}$$

where D_p is the diameter of the spherical or equivalent particle, $\epsilon = V_{free}/V_{total}$ is the porosity of the bed or column which is defined as the volume of voids divided by the volume of the bed or column, and $V_o = \dot{m}/(\rho A_c)$ is the superficial velocity which would result in the bed or column if no packing were present. The Ergun equation predicts the pressure drop (or flow) through porous media or packed columns quite well, see Fig. 3.8. It is widely used in the chemical process industries for modelling filtration beds, columns containing catalyst pellets, and percolation systems. Additional information on the flow of fluid through porous media may be found in Churchill (1987), Bear (1972), and McCabe et al. (1985).

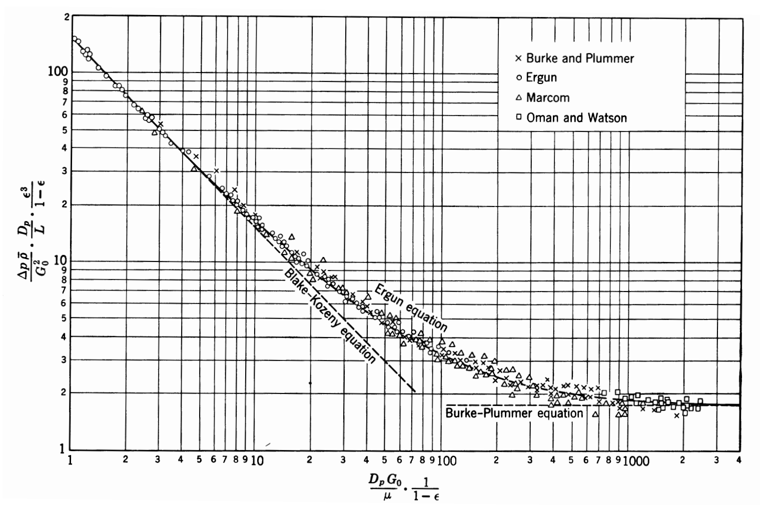


Fig. 3.8 - Flow Through Porous Media, From *Unit Operations in Chemical Engineering, McCabe et al.*, 1985.

Example 3.1

Examine the airflow through a heat sink which is composed of a number of channels formed by a series of plates. The system parameters are: fin height $H = 118$ mm, fin thickness $t = 1.25\ mm$, fin spacing $b = 2.5\ mm$, and fin length in direction of flow $L = 30\ cm$. The air properties may be taken to be $\rho = 1.2\ kg/m^3$, $\mu = 1.8 \times 10^{-5}$ $Pa \cdot s$. Calculate the pressure drop as a function of approach velocity V_∞. Include minor losses at the entrance and at the exit. What is the pressure drop when $V_\infty = 2$ m/s?

Example 3.2

A filtration component consists of a rectangular channel having dimensions $H = 10\ cm$, $W = 50\ cm$, and $L = 100\ cm$, containing carbon pellets of diameter 5 *mm*. Develop the pressure drop versus mass flowrate characteristic for the following porosities (due to packing arrangement): $\epsilon = 0.5, 0.7, 0.9$. The fluid properties may be taken to be $\rho = 1000\ kg/m^3$, $\mu = 1.0 \times 10^{-3}\ Pa \cdot s$. What is the pressure drop when $\dot{m} = 50\ kg/s$? Is it better to operate three filters in series or three in parallel. Why?

Example 3.3

Examine the electronics packaging enclosure described below. Nine circuit boards are placed in an enclosure with dimensions of $W = 50\ cm$, $H = 25\ cm$, and $L = 45\ cm$ in the flow direction. If the airflow required to adequately cool the circuit board array is 3 m/s over each board, determine the fan pressure required to overcome the losses within the system. Assume each board has an effective thickness of 5 mm, which accounts for the effects of the circuit board and components. You may further assume that the roughness of the boards is 2.5 mm. The air exhausts to atmospheric pressure. What effect would adding a louvered grill to the back of the cabinet have on the required fan pressure? How will your flow equations change? In your analysis include the effect of entrance and exit effects due the reduction in area. The density of air at 20 C is $\rho = 1.2\ kg/m^3$ and the viscosity is $\mu = 1.81 \times 10^{-5}\ Pa \cdot s$.

3.4 Pipe Networks

Most engineering systems are comprised of more than one section of pipe. In fact in most systems a complex network of piping is required to circulate the working fluid of a particular thermal system. These networks consist of series, parallel, and series-parallel configurations. We will examine each of these separately. However, before we proceed, a brief summary of the classification of piping system problems is necessary. This classification determines the type of solution which is obtainable when any two of the following principal design parameters are specified: head loss or Δp, volumetric flowrate Q, and pipe diameter D.

Pipe flow problems fall into three categories. In *Category I* problems the solution variable is the head loss or pressure drop Δp. The problem is specified such that the volumetric flow Q, the length of pipe L, the size or diameter D, are all known along with other parameters such as the pipe roughness and fluid properties. These types of problems yield a direct solution for the unknown variable Δp. In a *Category II* problem, the head loss (h or Δp) is specified and the volumetric flow Q is sought. Finally in a *Category III* problem, both the head loss and volumetric flow are specified, but the size or diameter of the pipe D is sought. *Category I* and *Category II* problems are considered analysis problems since the system is specified and only the flow is calculated. Whereas *Category III* problems are considered design problems, as the operating characteristics are known, but the size of the pipe is to be determined. Both *Category II* and *Category III* problems require an iterative approach in solution.

- TYPE I - $\Delta p \rightarrow Q$ or $\dot{m}, L, D$
- TYPE II - Q or $\dot{m} \rightarrow \Delta p, L, D$
- TYPE III - L or $D \rightarrow \Delta p, Q$ or $\dot{m}, D$ or L

Depending upon the nature of the flow (and solution process), it may be required to recompute other parameters such as the relative roughness at each iterative pass, since the ϵ/D ratio will change as the pipe diameter changes. However, with most modern computational software, we may solve "iterative" problems rather efficiently and need not resort to classic methods such as Gaussian elimination.

The simplest pipe networks are those that are either entirely composed of the series type of arrangement or the parallel type of arrangement as shown in Fig. 3.9. We will examine these two arrangements first and then proceed to the more complex series-parallel network.

3.4.1 Pipes in Series

The series flow arrangement is the simplest to analyze. In a series arrangement of pipes, the volumetric flow at any point in the system remains constant assuming the fluid is incompressible. Thus, for an arrangement of N pipes, the volumetric flow is given by

$$Q_1 = Q_2 = Q_3 = \cdots = Q_N = constant \tag{3.39}$$

or

$$V_1 A_1 = V_2 A_2 = V_3 A_3 = \cdots = V_N A_N = constant \tag{3.40}$$

The head loss in the system is the sum of the individual losses in each section of pipe. That is

$$\Delta p_{A \rightarrow B} = \Delta p_1 + \Delta p_2 + \Delta p_3 + \cdots + \Delta p_N \tag{3.41}$$

Note, that minor losses have not been accounted for in the above formulation. Pipe expansions and contractions account for significant losses at the joints. Further, care must be taken when analyzing the series system, as the pressure drop in Fig 3.9 (b) is not the same if the flow is reversed, i.e. from right to left, due to differences in the dynamic pressure at the inlet and outlet.

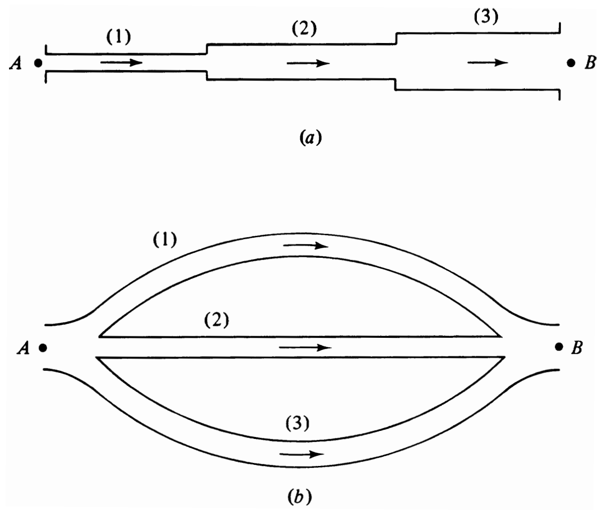


Fig. 3.9 - Series and Parallel Flow, From *Fluid Mechanics, White*, McGraw-Hill, 2000.

Example 3.4

Examine the series piping system consisting of three pipes each having a length $L = 1\ m$. The diameter of the first pipe is $D_1 = 0.02664\ m$, the diameter of the second pipe is $D_2 = 0.07792\ m$, and the diameter of the third pipe is $D_3 = 0.05252\ m$. Assume that the roughness $\epsilon = 0.0005\ m$ for each pipe. The fluid properties may be taken to be $\rho = 1000\ kg/m^3$, $\mu = 1.0 \times 10^{-3}\ Pa \cdot s$. Develop the pressure drop versus mass flowrate characteristic for the system assuming at first no minor losses and then include minor losses. What is the pressure drop when the mass flowrate is $\dot{m} = 20\ kg/s$? How significant are the minor losses relative to the piping losses?

3.4.2 Pipes in Parallel

Flow in parallel piping elements is also easy to analyze. In a parallel arrangement the total head loss or pressure drop across the system is constant. That is

$$\Delta p_1 = \Delta p_2 = \Delta p_3 = \cdots = \Delta p_N = constant \tag{3.42}$$

On the other hand, the volumetric flow through the system is the sum total of the individual flow in each pipe. That is

$$Q_{A \to B} = Q_1 + Q_2 + Q_3 + \cdots + Q_N \tag{3.43}$$

assuming an incompressible fluid. In many piping systems, parallel branches provide a means of flow bypassing, for diverting excess flow or for maintenance purposes. In a systems analysis, it is often desirable to develop the equivalent headloss curve as a function of the total flow through the branched network, rather than as a function of individual flows. We will address this issue later.

Example 3.5

Examine the parallel piping system consisting of three pipes each having a length $L = 1\ m$. The diameter of the first pipe is $D_1 = 0.02664\ m$, the diameter of the second pipe is $D_2 = 0.07792\ m$, and the diameter of the third pipe is $D_3 = 0.05252\ m$. Assume that the roughness $\epsilon = 0.0005\ m$ for each pipe. The fluid properties may be taken to be $\rho = 1000\ kg/m^3$, $\mu = 1.0 \times 10^{-3}\ Pa \cdot s$. Develop the pressure drop versus mass flowrate characteristic for each of pipes in the system and the pressure drop versus total mass flowrate characteristic, assuming no minor losses. What is the pressure drop when the total mass flowrate is $\dot{m} = 50\ kg/s$? At this flowrate what fraction of flow occurs in each branch?

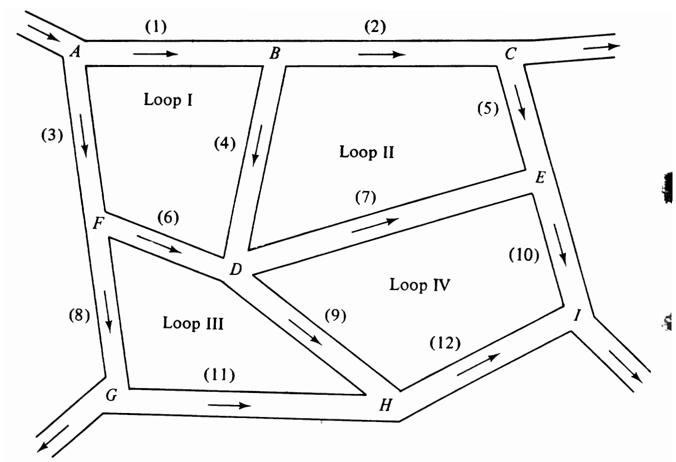


Fig. 3.10 - Pipe Flow Network, From *Fluid Mechanics, White*, McGraw-Hill, 2000.

3.4.3 Series-Parallel Networks

In a series-parallel pipe network as shown in Fig. 3.10, we must apply rules which are analogous to the analysis of an electric circuit. In Fig. 3.10 only shows the pipe network in 2-Dimensions. In reality, a pipe network is most often three dimensional. Thus, the elevations of each nodal point need to be considered when writing the extended Bernoulli equation. The following rules apply in any network of pipes:

1) The net flow into any junction is zero, $\sum Q_i = 0$
2) The net head loss around any loop must be equal to zero, $\sum \Delta p_i = 0$

Application of these rules leads to a complex set of equations which must be solved numerically. These are easily dealt with in most mathematical/numerical analysis programs. However, a method of hand calculation know as the Hardy-Cross method may also be applied, (Hodge and Taylor, 1999). This method is the basis for most computer software developed for analyzing piping systems. In this chapter we will address only planar systems, but the principles are easily applied to non-planar systems.

Example 3.6

Water flows in a pipe network (described by a sketch in class. The pipes forming the network have the following dimensions: $L_1 = 1777.7\ m$, $D_1 = 0.2023\ m$, $L_2 = 1524.4\ m$, $D_2 = 0.254\ m$, $L_3 = 1777.7\ m$,$D_3 = 0.3048\ m$, $L_4 = 914.6\ m$, $D_4 = 0.254\ m$, $L_5 = 914.6\ m$, and $D_5 = 0.254\ m$. If the mass flowrate entering the system is $\dot{m}_A = 50\ kg/s$ and $\dot{m}_B = 25\ kg/s$ and $\dot{m}_C = 25\ kg/s$ are drawn off the system at points B and C, compute the pressure drops and flow in each section of pipe. Ignore minor losses and assume that each junction is at the same elevation.

3.5 Manifolds and Distribution Networks

Manifolds are used to distribute a fluid within a mechanical system, usually on a small scale, according to specified requirements. Similarly, on a larger scale, distribution networks must direct fluid to a number of locations per some pre-selected criteria. Manifolds are often used in heat exchangers, automotive systems, and other fluid machinery. Distribution networks may be found in HVAC systems both for water and air flows, water supply systems, steam lines in a power plant, and process streams in a refinery to name a few. In both cases, a balancing of fluid flow is often desired such that each outlet receives essentially the same flow or some prescribed fraction of the total flow. The primary difference, is that manifold design is dominated by minor losses, while distribution networks are dominated by major losses.

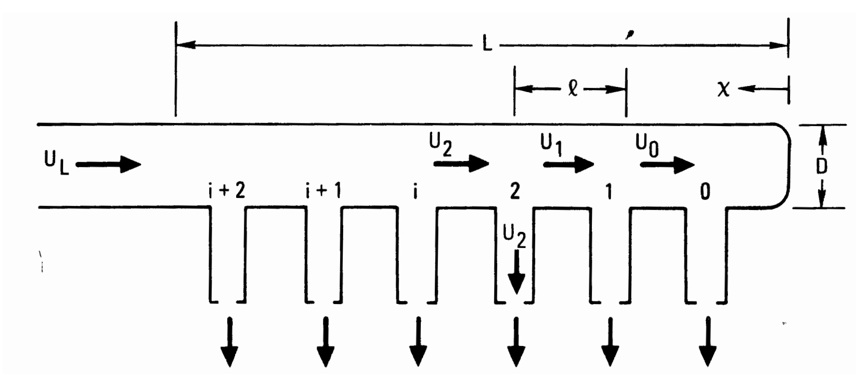


Fig. 3.11 - A Fluid Distribution Manifold, From *Applied Fluid Dynamics Handbook, Blevins*, Van Nostrand Reinhold, 1984.

Notation: A_i = area of the i pipe; D = pipe diameter; f = pipe friction factor for fully developed pipe flow; i = 1, 2, 3, an index denoting the branch supplying flow to the junction; j = 1, 2, 3, an index denoting a branch conveying fluid from the junction; K_{ij} = dimensionless loss coefficient; L = length of piping from point of measurement to junction; p = absolute static pressure; n = 1, 2, 3, an index used to denote which branch's velocity is used in constructing loss term; U = flow velocity in a pipe, averaged over pipe cross section; r = radius of rounding (see Fig. 6-20); θ = angle of branch (degrees); ρ = fluid density. Results are for turbulent flow, $UD/\nu > 10^4$. See Table 3-1 for consistent sets of units. See Fig. 6-20.

Total Pressure Loss, $p_i - p_j + \frac{1}{2}\rho(U_i^2 - U_j^2) = \frac{1}{2}\rho U_n^2 K_{ij} + \frac{1}{2}\rho U_i^2 \frac{f_i L_i}{D_i} + \frac{1}{2}\rho U_j^2 \frac{f_j L_j}{D_j}$

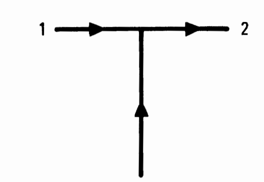
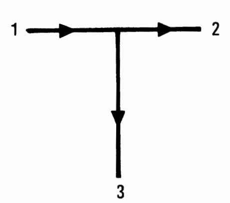
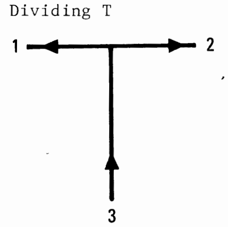
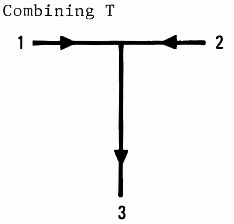
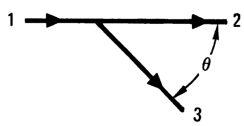
Description	Pipe Areas	i	j	n	Loss Coefficient, K_{ij}
1. Combining T  $0 \le r/D \le 0.5$; $0 \le U_3/U_2 \le 1$	$A_1 = A_2$ Data for $A_3 = A_2$ approximate for $A_3 < A_2$.	1	2	2	$K_{12} = 0.045 + \left[1.38 - 1.94\left(\frac{r}{D}\right)^{1/2} + 1.34\frac{r}{D}\right]\frac{U_3}{U_2} - \left[0.90 - 0.95\left(\frac{r}{D}\right)^{1/2} + 1.23\left(\frac{r}{D}\right)\right]\left(\frac{U_3}{U_2}\right)^2$
		3	2	2	$K_{32} = 1.09 - 0.80\left(\frac{r}{D}\right)^{1/2} - \left[0.53 + 1.27\left(\frac{r}{D}\right)^{1/2} - 1.86\frac{r}{D}\right]\frac{U_1}{U_2} - \left[1.48 - 2.28\left(\frac{r}{D}\right)^{1/2} + 1.80\frac{r}{D}\right]\left(\frac{U_1}{U_2}\right)^2$ Ref. 6-106
2. Dividing T  $0 \le r/D \le 0.5$; $0 \le U_3/U_2 \le 1$	$A_1 = A_2$ Data for $A_3 = A_2$ approximate for $A_3 < A_2$. See text for expressions for $A_3 < A_2$ and $r/D = 0$.	1	2	1	$K_{12} = \begin{cases} 1.55\left(0.22 - \frac{U_3}{U_1}\right)^2 - 0.03; & 0 \le \frac{U_3}{U_1} \le 0.22 \\ 0.65\left(\frac{U_3}{U_1} - 0.22\right)^2 - 0.03; & 0.2 \le \frac{U_3}{U_1} \le 1. \end{cases}$
		1	3	1	$K_{13} = 0.99 - 0.23\left(\frac{r}{D}\right)^{1/2} - \left[0.82 + 0.29\left(\frac{r}{D}\right)^{1/2} + 0.30\frac{r}{D}\right]\frac{U_3}{U_1} + \left[1.02 - 0.64\left(\frac{r}{D}\right)^{1/2} + 0.76\frac{r}{D}\right]\left(\frac{U_3}{U_1}\right)^2$ Ref. 6-106.
3. Dividing T 	$U_1 = U_2$ $A_1 = A_2 = A_3$	3	1	3	$K_{31} = 0.59 + \left[1.18 - 1.84\left(\frac{r}{D}\right)^{1/2} + 1.16\frac{r}{D}\right]\frac{U_1}{U_3} - \left[0.68 - 1.04\left(\frac{r}{D}\right)^{1/2} + 1.16\frac{r}{D}\right]\left(\frac{U_1}{U_3}\right)^2$; $0 \le \frac{r}{D} \le 0.5$; $0.2 \le \frac{U_1}{U_3} \le 0.8$ Ref. 6-106.
4. Combining T 	$U_1 = U_2$ $A_1 = A_2 = A_3$	1	3	3	$K_{13} = 1.19 - 1.16\left(\frac{r}{D}\right)^{1/2} + 0.46\frac{r}{D} - 1.73\left(1 - \frac{r}{D}\right)\frac{U_2}{U_3} + \left(1.34 - 1.69\frac{r}{D}\right)\left(\frac{U_2}{U_3}\right)^2$; $0 \le \frac{r}{D} \le 0.5$; $0 \le \frac{U_2}{U_3} \le 1$ Ref. 6-106.

Fig. 3.12a - Branching Loss Factors, From *Applied Fluid Dynamics Handbook, Blevins*, Van Nostrand Reinhold, 1984.

Total Pressure Loss, $p_i - p_j + \frac{1}{2}\rho(U_i^2 - U_j^2) = \frac{1}{2}\rho U_n^2 K_{ij} + \frac{1}{2}\rho U_i^2 \frac{f_i L_i}{D_i} + \frac{1}{2}\rho U_j^2 \frac{f_j L_j}{D_j}$

Description	Pipe Areas	i	j	n	Loss Coefficient, K_{ij}
5. Lateral Dividing Branch	$A_1 = A_2$ $\frac{A_3}{A_1} \le 1.0$	1	3	1	



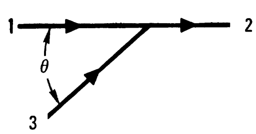
$K_{13}^{(a)(b)}$

$\frac{U_3}{U_1}$	θ(deg) 90, $\frac{r}{D} = 0$	90, $\frac{r}{D} = 0.1$	60, $\frac{r}{D} = 0$	60, $\frac{r}{D} = 0.1$	45, $\frac{r}{D} = 0$	45, $\frac{r}{D} = 0.1$
0.0	1.0	1.0	1.0	1.0	1.0	1.0
0.5	1.1	1.1	0.7	0.6	0.5	0.5
1.0	1.5	1.3	0.8	0.7	0.5	0.5
1.5	2.1	1.7	1.3	1.0	0.9	0.7
2.0	2.8	2.3	2.0	1.5	1.5	1.2
2.5	3.8	2.8	2.8	2.1	2.4	1.7
3.0	5.0	3.6	3.9	2.8	3.3	2.5
3.5	6.5	4.6	5.0	3.7	4.4	3.2

[a] Results are nearly independent of A_3/A_1 for $0.3 \le A_3/A_2 \le 1.0$.

[b] For $\theta = 90°$ and $A_1 = A_3$, see frame 2 for more accurate result.

Description	Pipe Areas	i	j	n	Loss Coefficient, K_{ij}
		1	2	3	K_{12} increases approximately as $(U_3/U_1)^2$ from $K_{12} \approx 0$ at $U_3 = 0$ to $K_{12} \approx 0.5$ when all flow is diverted into branch (3) and is nearly independent of θ or r/D. Ref. 6-107.
6. Lateral Combining Branch	$A_1 = A_2$ Data for $A_3 = A_2$ approximate for $A_3 < A_2$.	3 1	2 2	2 2	



$\frac{U_3}{U_2}$	K_{32} θ(deg) 60	K_{32} 45	K_{32} 30	K_{12} θ(deg) 60	K_{12} 45	K_{12} 30
0	-1.0	-0.95	-1.0	0.0	0.	0
0.1	-0.62	-0.57	-0.63	0.18	0.08[a]	0.17
0.2	-0.26	-0.37	-0.35	0.32	0.14[a]	0.29
0.3	-0.10	-0.18	-0.10	0.42	0.19[a]	0.35
0.4	0.28	-0.02	0.16	0.48	0.18[a]	0.36
0.5	0.50	0.12[a]	0.27	0.50	0.13[a]	0.32
0.6	0.68	0.20[a]	0.31	0.48	0.08[a]	0.25
0.7	0.84	0.28[a]	0.40	0.42	-0.03	0.10
0.8	0.92	0.32[a]	0.45	0.32	-0.20	-0.15
0.9	0.99	0.33[a]	0.40	0.18	-0.35	-0.45
1.0	1.00	0.32[a]	0.27	0.0	-0.57	-0.75

[a] Ref. 6-39, p. 260 gives values about twice as large for same conditions.

Ref. 6-39, p. 260 (30°, 60°); Ref. 6-42, p. 56 (45°). Losses are somewhat higher for $A_3 < A_1$. See text.

Fig. 3.12b - Branching Loss Factors, From *Applied Fluid Dynamics Handbook, Blevins*, Van Nostrand Reinhold, 1984.

Total Pressure Loss, $p_i - p_j + \frac{1}{2}\rho(U_i^2 - U_j^2) = \frac{1}{2}\rho U_n^2 K_{ij} + \frac{1}{2}\rho U_i^2 \frac{f_i L_i}{D_i} + \frac{1}{2}\rho U_j^2 \frac{f_j L_j}{D_j}$

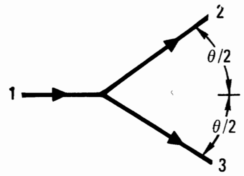
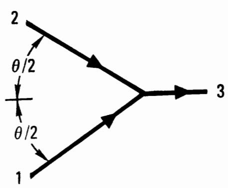
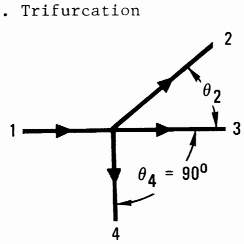
Description	Pipe Areas	i	j	n	Loss Coefficient, K_{ij}
7. Dividing Y  (conical junction)	$A_2 = A_3 = \frac{A_1}{2}$ $D_2 = D_3 = \frac{D_1}{\sqrt{2}}$	1	2	1	Table 7 below. Ref. 6-108.
8. Combining Y 	$A_1 = A_2 = \frac{A_3}{2}$ $D_1 = D_2 = \frac{D_3}{\sqrt{2}}$	2	3	3	Table 8 below. Ref. 6-39, p. 301.
9. Trifurcation  $\theta_2 \le 90°$, $\theta_3 = 0°$	$A_1 = A_3 = A_4$ $A_2 \le A_1$ (formula may apply to other trifurcations)	1	j	1	$K_{1j} = 0.96 \sin^2\theta_j + \alpha_j\left(\cos\theta_j - \frac{U_j}{U_1}\right)^2 + \beta_j \frac{U_j}{U_1}$, where $\alpha_j = 0.0227\,\theta_j \cos\theta_j + 1.2 \sin\theta_j \sin\lvert 60 - \theta_j\rvert$ $\beta_j = \left[0.00698\,(45 - \theta_j) + 0.075\frac{A_j}{A_1} + 0.0262\,\theta_j\right] \sin(75 - \theta_j)$ (θ in degrees, $\theta \le 90°$) Ref. 6-109.

Table 7: K_{12}

U_2/U_1	θ(deg) 90	60	45
0	0.54	0.55	0.53
0.2	0.43	0.44	0.41
0.4	0.30	0.29	0.25
0.6	0.22	0.17	0.15
0.8	0.16	0.08	0.08
1.0	0.14	0.06	0.04
1.2	0.13	0.04	0.03
1.4	0.14	0.05	0.07
1.6	0.17	0.07	0.14
1.8	0.22	0.13	0.27
2.0	0.32	0.29	0.52

Table 8: K_{13}

U_2/U_3	θ(deg) 90	60	30
0	-1.30	-2.05	-2.56
0.2	-0.93	-1.51	-1.89
0.4	-0.55	-1.00	-1.30
0.6	-0.16	-0.53	-0.77
0.8	0.20	-0.10	-0.30
1.0	0.56	0.28	0.10
1.2	0.92	0.69	0.41
1.4	1.26	0.91	0.67
1.6	1.61	1.09	0.85
1.8	1.95	1.37	0.97
2.0	2.30	1.55	1.04

Fig. 3.12c - Branching Loss Factors, From *Applied Fluid Dynamics Handbook, Blevins*, Van Nostrand Reinhold, 1984.

The key to analyzing these systems is that most often the fluid flows from a single point at a known or calculated pressure, to several other locations which are all at some lower pressure. This constitutes a series-parallel network which usually results in a series of difference equations. The underlying principle is that the *total* pressure drop between inlet and any exit is the same. Thus the size of the system is to be determined for balancing or equalizing flow out of each exit port. However, the flow at any point along the path of interest is *not* constant. Each loss in the system must be calculated at the flow rate which flows through the segment of interest. The total pressure drop for each section is then calculated and summed over each flow path. This is best illustrated by means of a sample calculation. Fig. 3.11 shows a typical fluid splitting manifold. If the flow is reversed then a collection manifold is obtained. Fig. 3.12 (a-c) provide useful K-factors for branching flows.

Example 3.7

Examine the system given below. The water distribution system is to be designed to give equal mass flow rate to each of the two locations, which are not of equal distance from the source. In order to achieve this, two pipes of different diameter are used. Determine the size of the longer pipe which yields the same mass flow rate. You may assume that all of the kinetic energy is lost at the terminations of the pipeline and that the pressure is atmospheric. In your solution

- Develop the basic equations for each branch of the system
- Determine the required diameter of the longer pipe
- Assume K=1.5 for the junction connection

The density of water at 20 C is $\rho = 1000\ kg/m^3$ and the viscosity is $\mu = 1 \times 10^{-3}$ $Pa \cdot s$.

Example 3.8

You are to design an air distribution system having the following layout: main line diameter $D = 50\ cm$ and four equally spaced branch lines having diameter $d = 30$ cm. Each branch line is to have the same air flow. To achieve this, you propose using a damper having a well defined variable loss coefficient, to control the flow in each branch. Determine the value of the loss coefficient for each damper, such that the system is balanced. Each section of duct work is 5 m in length. A total flow of 10 m^3/s is to be delivered by a fan. In your solution consider the minor losses at the junctions $K = 0.8$ and exits $K = 1.0$. What fan pressure is required? Assume air properties to be $\rho = 1.1\ kg/m^3$, and $\mu = 2 \times 10^{-5}\ Pa \cdot s$.

Example 3.9

You are to analyze the flow through a flat plate solar collector system as shown in class. The system consists of a series of pipes connected to distribution and collection manifolds. Make any necessary assumptions.

3.6 Two Phase Flow Models

We conclude the discussion of flow analysis by considering the important topic of two phase flow in pipes. Two phase flows occur in steam plants as the working fluid passes through the boiler and condenser, in refrigeration systems as the refrigerant passes through the condenser and evaporator, and in oil and gas operations as a mixture of gas and oil during production and separation processes. In many applications more than two phases may be present. Particularly in chemical processes where a solid phase may also be present in addition to the gas and liquid phases. In this course we will only consider two phase liquid/gas systems. Further, the flow may be considered to be adiabatic or non-adiabatic. Non-adiabatic flows occur during phase change operations. Finally, there is a distinction between horizontal, vertical up flow, and vertical down flow, refer to Figs. 3.13-3.16 given below.

In order to undertake a two phase flow analysis a number of important concepts and definitions need to be addressed. First, the void fraction is defined as the ratio of the gas volume to the total mixture volume in an elemental control volume. That is

$$\alpha_g = \frac{V_g}{V} = \frac{A_g}{A} \tag{3.44}$$

We may also define the void fraction in terms of the liquid such that

$$\alpha_l = \frac{V_l}{V} = \frac{A_l}{A} \tag{3.45}$$

These are related by means of the following relationship:

$$\alpha_g + \alpha_l = 1 \tag{3.46}$$

Next we must define the mass flux G such that

$$G = \rho U = \frac{\dot{m}}{A} \tag{3.47}$$

Now for each phase we may define the mass flux in terms of the superficial velocities, that is the velocity of each phase in the pipe if each phase occupied the total cross-sectional area. These are:

$$G_g = \rho_g U_g \tag{3.48}$$

and

$$G_l = \rho_l U_l \tag{3.49}$$

The actual phase velocities are defined using the void fraction:

$$u_g = \frac{U_g}{\alpha_g} = \frac{G_g}{\rho_g \alpha_g} \tag{3.50}$$

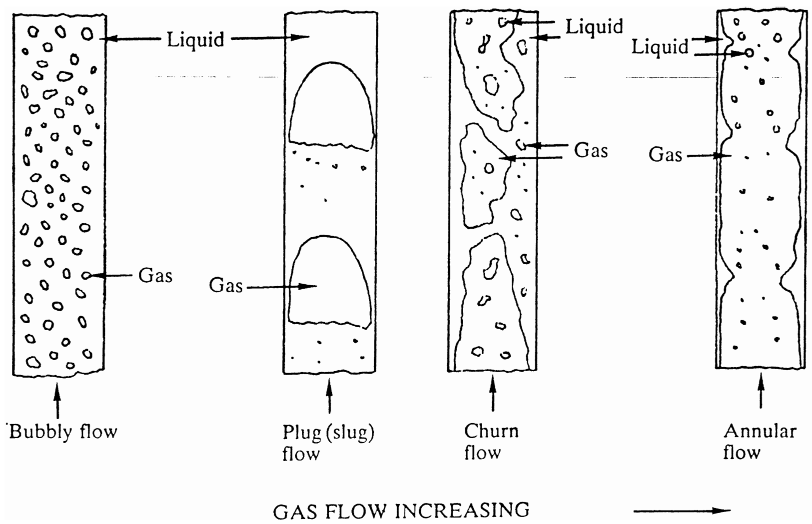


Fig. 3.13 - Two Phase Flow Patterns in a Vertical Pipe, From *Two Phase Flow and Heat Transfer, Whalley*, Oxford, 1996.

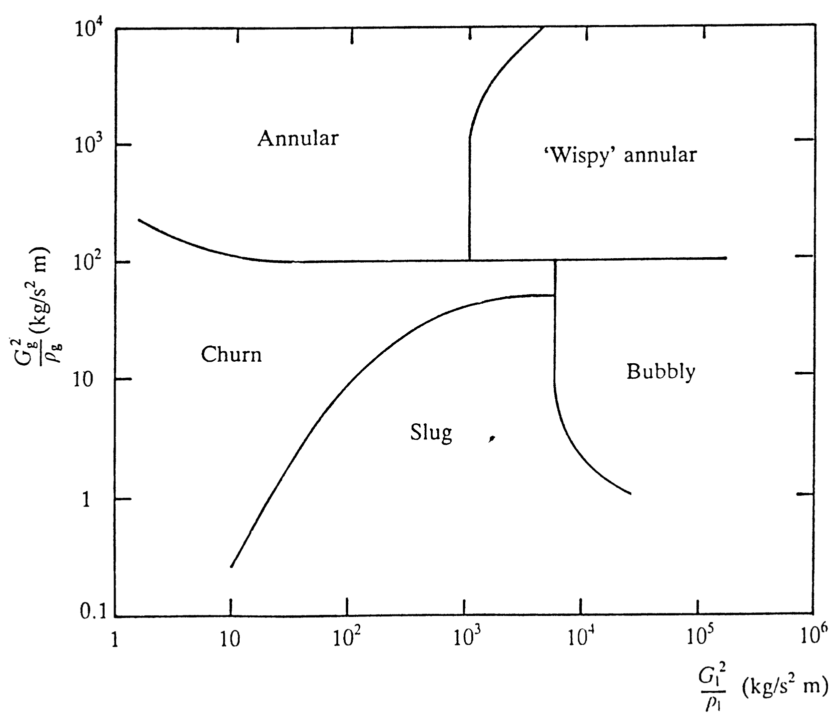


Fig. 3.14 - Two Phase Flow Map for Vertical Up Pipe Flow, From *Two Phase Flow and Heat Transfer, Whalley*, Oxford, 1996.

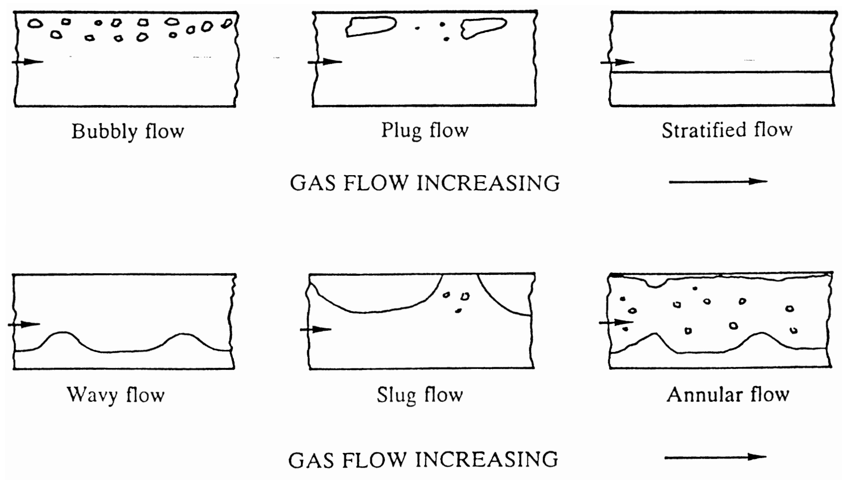


Fig. 3.15 - Two Phase Flow Patterns in a Horizontal Pipe, From *Two Phase Flow and Heat Transfer, Whalley*, Oxford, 1996.

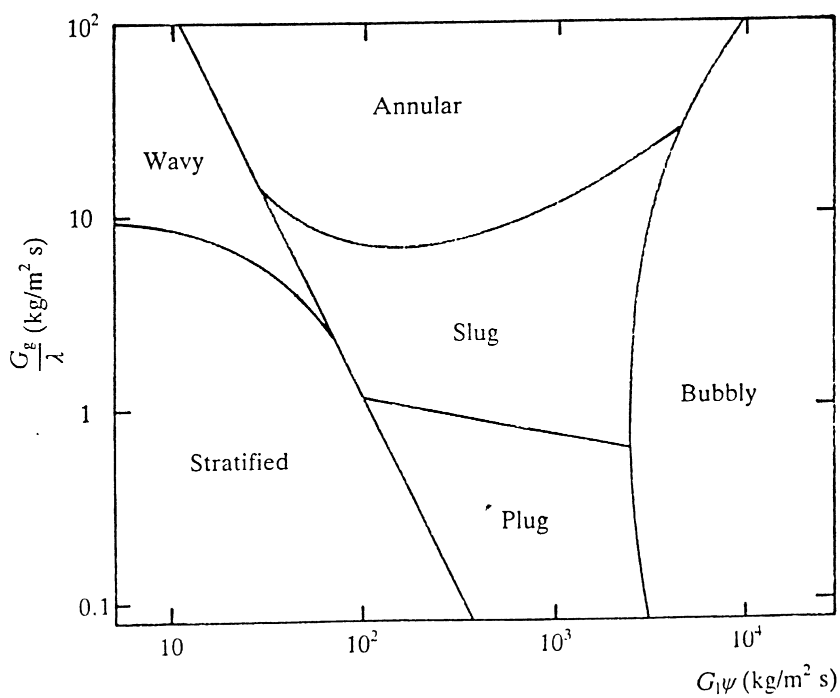


Fig. 3.16 - Two Phase Flow Map for Horizontal Pipe Flow, From *Two Phase Flow and Heat Transfer, Whalley*, Oxford, 1996.

and

$$u_l = \frac{U_l}{\alpha_l} = \frac{G_l}{\rho_l \alpha_l} \tag{3.51}$$

Finally, the quality of the flow is defined as

$$x = \frac{G_g}{G_g + G_l} \tag{3.52}$$

It takes a value of 0 for liquid only flow, and a value of 1 for gas only flow.

In a two phase flow, the total pressure gradient on an elemental control volume may be obtained (assuming a homogeneous flow):

$$-\frac{dp}{dz} = \underbrace{\frac{4\tau_w}{D_h}}_{friction} + \underbrace{\overline{\rho} g \sin(\theta)}_{gravitational} + \underbrace{G^2 \frac{d}{dz}\left(\frac{1}{\overline{\rho}}\right)}_{accelerational} \tag{3.53}$$

The frictional component is most often the largest, unless there is significant change in density of the mixture such as during a non-adiabatic flow. It accounts for the viscous action of the fluid mixture on the duct walls and for interphase effects. The gravitational term accounts for the elevation change the mixture experiences. It can easily be accounted for if teh void fraction is known. Finally, the accelerational term accounts for the change in pressure due to changes in density. It is only important when phase change occurs or when the density changes along the flow path such as during injection processes. The mean density may be determined using volume averaging for a *homogeneous* flow, i.e.,

$$\overline{\rho} = \alpha_g \rho_g + \alpha_l \rho_l = \left[\frac{x}{\rho_g} + \frac{1-x}{\rho_l}\right]^{-1} \tag{3.54}$$

In two phase flow analysis, many approaches have been adopted for prediction of the frictional pressure gradient. These include homogeneous flow models and separated flow models. In a homogeneous flow model, the gas and liquid are assumed to have the same velocity, i.e., a single mixture velocity. While in a separated flow model, they can have different flow velocities. In general separated flow models are more accurate as they rely on the actual phase velocities. However, due the complex nature of two phase flows, often accuracy of only 40 percent or better can be obtained.

Most two phase flow models use a concept referred to as a two phase flow multiplier. That is a special correction factor is calculated which corrects the single phase flow pressure gradient for the gas mass flux G_g or liquid mass flux G_l, i.e.,

$$\phi_g^2 = \frac{(dp/dz)}{(dp/dz)_g} \tag{3.55}$$

or

$$\phi_l^2 = \frac{(dp/dz)}{(dp/dz)_l} \tag{3.56}$$

More often, though, the models use a gas only or a liquid only multiplier. That is, they assume that either gas or liquid only flow with the total mass flux G, i.e.,

$$\phi_{go}^2 = \frac{(dp/dz)}{(dp/dz)_{go}} \tag{3.57}$$

or

$$\phi_{lo}^2 = \frac{(dp/dz)}{(dp/dz)_{lo}} \tag{3.58}$$

These single phase flow pressure gradients are computed using traditional methods such as the Swamee and Jain equation or the Churchill equation discussed earlier.

We now conclude with a discussion of three different separated flow models which are widely used in industry. These are: the Lockhart-Martinelli model, the Chisholm model, and the Friedel model. These models were developed for tubes and pipes having diameters typically greater than about 1/2 centimeter.

Lockhart-Martinelli Model

In the Lockhart-Martinelli model, the two phase multiplier is defined using the parameter

$$X^2 = \frac{(dp/dz)_l}{(dp/dz)_g} \tag{3.59}$$

The two phase multipliers are then computed using

$$\phi_l^2 = 1 + \frac{C}{X} + \frac{1}{X^2} \tag{3.60}$$

or

$$\phi_g^2 = 1 + CX + X^2 \tag{3.61}$$

where C is tabulated below. There are four possible flow scenarios depending on the type of flow each phase experiences, i.e. laminar or turbulent. This is determined using the appropriate Reynolds numbers for each phase. In this case since the two phase flow multiplier is based on the liquid or gas phase mass flux, then so to are the Reynolds numbers.

Liquid	Gas	C
Turbulent	Turbulent	20
Laminar	Turbulent	12
Turbulent	Laminar	10
Laminar	Laminar	5

Chisholm Model

The Chisholm model is a little more complex, but provides impreoved accuracy. It takes the following form:

$$\phi_{lo}^2 = 1 + (Y^2 - 1)\left[Bx^{(2-n)/2}(1-x)^{2-n)/2} + x^{(2-n)}\right] \tag{3.62}$$

where

$$\begin{array}{ll} B = 55/G^{1/2} & 0 < Y < 9.5 \\ B = 520/(Y \cdot G^{1/2}) & 9.5 < Y < 28 \\ B = 15000/(Y^2 \cdot G^{1/2}) & Y > 28 \end{array} \quad (3.63)$$

and

$$Y^2 = \frac{(dp/dz)_{go}}{(dp/dz)_{lo}} \quad (3.64)$$

The exponent n takes the value $n = 1$ for laminar flow and $n = 1/4$ for turbulent flow when calculating the liquid only Reynolds number.

Friedel Model

The Friedel model takes the following form for the two phase flow multiplier

$$\phi_{lo}^2 = E + \frac{3.24F \cdot H}{Fr^{0.045}We^{0.035}} \quad (3.65)$$

where

$$E = (1-x)^2 + x^2 \left(\frac{\rho_l f_{go}}{\rho_g f_{lo}} \right) \quad (3.66)$$

and

$$F = x^{0.78}(1-x)^{0.24} \quad (3.67)$$

and

$$H = \left(\frac{\rho_l}{\rho_g} \right)^{0.91} \left(\frac{\mu_g}{\mu_l} \right)^{0.19} \left(1 - \frac{\mu_g}{\mu_l} \right)^{0.7} \quad (3.68)$$

The Froude and Weber numbers are defined as:

$$Fr = \frac{G^2}{gD_h\overline{\rho}^2} \quad (3.69)$$

$$We = \frac{G^2 D_h}{\overline{\rho}\sigma} \quad (3.70)$$

where σ is the surface tension of the liquid and $g = 9.81$ is the gravitational constant.

In general, the Chisholm and Friedel models are the best with Friedel providing the greatest accuracy as it is based on a very large data set comprising some 25,000 data points. Note if the Friedel model is used with two immiscible fluids, the less viscous phase must be taken as the "gas", i.e. $\mu_g < \mu_l$.

The above models are recommended for use under the following conditions [Hetsroni, 1982]:

- For $\mu_l/\mu_g < 1000$, the Friedel model should be used.
- For $\mu_l/\mu_g > 1000$ and $\dot{m} > 100$, the Chisholm model should be used.

- $\mu_l/\mu_g > 1000$ and $\dot{m} < 100$, the Lockhart-Martinelli should be used.

In this regard, it is recommended to using bounding models to provide some reasonable level of certainty in analysis. These simple bounding models are discussed next.

Bounds on Two Phase Flow Calculations

To alleviate the issue of variations between the various two phase flow models, Awad and Muzychka (2005), developed simple bounding models which determine the reasonable expected range of two phase flow frictional pressure drop and void fraction. These models provide the upper, lower, and mean values of these parameters for *turbulent* two phase flow in pipes.

Frictional pressure gradient can be bounded such that:

$$\left(\frac{dp}{dz}\right)_l < \overline{\frac{dp}{dz}} < \left(\frac{dp}{dz}\right)_u \tag{3.71}$$

where the lower and upper bounds are given by:

$$\left(\frac{dp}{dz}\right)_l = \frac{0.158 G^{7/4}(1-x)^{7/4}\mu_l^{1/2}}{\rho_l D^{5/4}} \left[1 + \left(\frac{x}{1-x}\right)^{0.7368} \left(\frac{\rho_l}{\rho_g}\right)^{0.4211} \left(\frac{\mu_g}{\mu_l}\right)^{0.1053}\right]^{2.375}$$

$$\left(\frac{dp}{dz}\right)_u = \frac{0.158 G^{7/4}(1-x)^{7/4}\mu_l^{1/2}}{\rho_l D^{5/4}} \left[1 + \left(\frac{x}{1-x}\right)^{0.4375} \left(\frac{\rho_l}{\rho_g}\right)^{0.25} \left(\frac{\mu_g}{\mu_l}\right)^{0.0625}\right]^{4}$$

and, the mean obtained from:

$$\overline{\frac{dp}{dz}} = \frac{1}{2}\left[\left(\frac{dp}{dz}\right)_l + \left(\frac{dp}{dz}\right)_u\right] \tag{3.72}$$

The void fraction can also be modelled in a similar manner such that:

$$\alpha_l < \overline{\alpha} < \alpha_u \tag{3.73}$$

where the lower and upper bounds are given by:

$$\alpha_l = \frac{1}{1 + \left[\left(\frac{1-x}{x}\right)^{0.875} \left(\frac{\rho_g}{\rho_l}\right)^{0.5} \left(\frac{\mu_l}{\mu_g}\right)^{0.125}\right]^{0.84}}$$

$$\alpha_u = \frac{1}{1 + 0.28\left[\left(\frac{1-x}{x}\right)^{0.875} \left(\frac{\rho_g}{\rho_l}\right)^{0.5} \left(\frac{\mu_l}{\mu_g}\right)^{0.125}\right]^{0.71}}$$

and, the mean obtained from

$$\overline{\alpha} = \frac{1}{2}\left[\alpha_l + \alpha_u\right] \tag{3.74}$$

The above expressions allow one to compute the reasonable expected limits in a two phase pipe flow. Since they are based on experimental data which have considerable spread due to experimental errors and the general complexity of the flow, the bounds can be as large as ±50% for pressure drop and ±20% for void fraction. Unfortunately this is the nature of two phase flow.

Example 3.10

Air and water flow in a three inch diameter pipe. The mass flux is $G = 500\ kg/sm^2$ and the quality is $x = 0.1$. Determine the frictional pressure gradient required to move the flow using the Lockhart-Martinelli, Chisolm, and Friedel models. Assume $T = 30\ C$. Next use the bounded modelling approach and compare all of the results.

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